
The Effects of Metacognitive Instruction on Solving Mathematical Authentic Tasks

Author(s): Bracha Kramarski, Zemira R. Mevarech, Marsel Arami

Source: *Educational Studies in Mathematics*, Vol. 49, No. 2 (2002), pp. 225-250

Published by: Springer

Stable URL: <http://www.jstor.org/stable/3483076>

Accessed: 04/03/2010 20:30

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=springer>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Springer is collaborating with JSTOR to digitize, preserve and extend access to *Educational Studies in Mathematics*.

<http://www.jstor.org>

THE EFFECTS OF METACOGNITIVE INSTRUCTION ON SOLVING MATHEMATICAL AUTHENTIC TASKS

ABSTRACT. The present study investigates the differential effects of cooperative-learning with or without metacognitive instruction on lower and higher achievers' solutions of mathematical authentic tasks. Participants were 91 seventh graders who studied in three classrooms. Data were analyzed by using qualitative and quantitative methods. Results indicated that students who were exposed to the metacognitive instruction within cooperative learning (COOP+META) significantly outperformed their counterparts who were exposed to cooperative learning with no metacognitive instruction (COOP). The positive effects of COOP+META were observed on both authentic and standard tasks. In addition, the findings show the positive effects of COOP+META method on lower and higher achievers. The practical implications of the study are discussed.

KEY WORDS: authentic tasks, cooperative learning, metacognition, problem solving

INTRODUCTION

The new reforms for teaching mathematics redirect teachers to focus on solving authentic tasks (NCTM, 2000; Cai, 2000). This mandate is rooted in at least three sources. First, today's economy requires highly skilled workers who are able to apply their mathematical knowledge to solve authentic tasks for which the solution method is not known in advance (e.g., Darling-Hammond, 1992). Second, some mathematics educators (e.g., Cobb, 1994) emphasize the importance of using authentic tasks because knowledge construction is likely to occur when students have to deal with rich information and resolve cognitive conflicts, rather than applying ready-made algorithms for solving standard tasks (Prawat, 1998). Finally, from a motivational perspective, providing challenging tasks that are relevant to the students' world and daily life has the potential to increase students' interest in mathematics, which in turn may enhance achievement (e.g., Cai, 2000; Hattie, Biggs and Purdie, 1996; Hoek, van den Eden and Terwel, 1999). The main purpose of the present study is, therefore, to investigate instructional methods that have the potential to enhance students' ability to solve mathematical authentic tasks.



MATHEMATICAL AUTHENTIC TASKS

Mathematical Authentic tasks are those which portray common contexts and for which there are no ready-made algorithms (Forman and Steen, 2000). For example, teenagers in Western countries are used to buying pizzas and refreshments for parties. An authentic task may ask students to analyze data that would enable them to decide which is the best offer for ordering pizza for a party, given the budget, number of guests, the prices at different restaurants, and the factors that determine the price of a pizza (e.g., size, number of slices, and number and kinds of supplements). Forman and Steen (2000) as well as others (NCTM, 2000; Cai, 2000; Lampert, 1990; Rose and Schuncke, 1997; Silver, 1994) further emphasize that authentic tasks employ realistic data that are often incomplete or inconsistent, avoid artificial worksheets, provide rich information about the described situation, include complex mathematical data, can be approached in different ways, are based on a wide range of mathematical knowledge and mathematical skills, and often ask solvers to use different representations in their solutions.

It should be noted that being an authentic task is not a property of a problem but of the relation between the problem solver to the problem. The pizza problem described above may be authentic for youngsters who are used to going to restaurants and comparing prices, but it may be inauthentic for students living in countries where such activities are not common.

Authentic tasks are rarely presented in mathematics classrooms. Instead, the standard tasks customarily used by teachers are those which describe simplified situations involving some quantitative information and for which there are ready-made algorithms that students have to apply in order to solve the problem. Word problems are examples of standard tasks commonly presented in mathematics textbooks.

Many students, lower as well as higher achievers, face difficulties in solving authentic tasks (e.g. Verschaffel, Greer and De Corte, 2000; Kramarski, Mevarech and Liberman, 2001). The difficulties are raised at all stages of the solution process from the very first stage of understanding what the problem is all about, through planning the solution process and selecting appropriate strategies, to reflecting on the solution and deciding whether or not it makes sense (Verschaffel et al., 2000). Some students, particularly lower achievers, do not see the task as a whole, and thus focus on only parts of the task (e.g., Lester, 1994). According to Cardel-Elawar, (1995), Anderson (1990) and Fry (1987) lower achievers: read rapidly the task at the expense of fully comprehending it, do not recognize that there might be more than one right way to solve the task, and are uncertain

about how to calculate and verify the solution. Verschaffel et al. (2000) indicated that lower achievers have difficulties in reorganizing information and in distinguishing between relevant and irrelevant information. Higher achievers face different difficulties. They give up easily because ready-made algorithms are not available for solving the authentic task, and they have difficulties in bridging between what they know about standard tasks and the novel, authentic tasks (Anderson, 1990; Fry, 1987; Verschaffel, et. al., 2000).

Although authentic tasks are important, little is known at present on how to enhance students' ability to solve such tasks. Whereas much research has focused on enhancing mathematical achievement under innovative instructional methods, and in particular under cooperative learning (e.g., Cardell-Elawar, 1995; Webb and Farivar, 1994), experimental or quasi-experimental studies seldom utilized authentic tasks as a measure of outcomes. This is not surprising given the fact that solving authentic tasks is time consuming and therefore teachers have reservations about introducing such tasks either in the ongoing instruction or in testing situations. Since many of the difficulties associated with solving authentic tasks lie in students' inability to control, monitor, and reflect on their solution processes, the present study examines the extent to which metacognitive instruction facilitates the solution of authentic tasks.

METACOGNITIVE INSTRUCTION IN MATHEMATICS CLASSROOMS

Since the late 1970s, metacognition received a lot of attention in the educational literature (e.g., Brown, Bransford, Ferrara, Campione, 1983, Flavell, 1978; Jacobs and Paris, 1987; Swanson, 1990). Metacognition is defined as "the knowledge and control one has over one's thinking and learning activities" (Swanson, 1990, p. 306). Following Flavell (1978), researchers investigated many aspects of metacognition, including: the development of metacognition in various age groups (King, 1989, 1991, 1994), the activation of metacognitive processes in different contexts such as reading (e.g., Cross and Paris, 1988), mathematics (e.g., Schoenfeld, 1987), and memory (e.g., Pressley, Borkowski and Sullivan, 1985), and the relationships between metacognition and other cognitive functions (e.g., Swanson, 1990). But only lately, however, researchers (e.g., Schoenfeld, 1987; Mayer, 1987; Lester, Garofalo and Kroll, 1989; Cardell-Elawar, 1995, Mevarech and Kramarski, 1997) have started to design instructional methods that are based on training students to activate metacognitive processes during the solution of mathematical tasks.

A major common element of metacognitive instruction is training students who work in small groups to reason mathematically by formulating and answering a series of self-addressed metacognitive questions. These questions focus on: (a) *comprehending* the problem (e.g., "What is the problem all about?"); (b) constructing *connections* between previous and new knowledge (e.g., "What are the similarities/differences between the problem at hand and the problems you have solved in the past? and why?"; (c) using *strategies* appropriate for solving the problem (e.g., "What are the strategies/tactics/principles appropriate for solving the problem and why?"; and in some studies also (d) *reflecting* on the processes and the solution (e.g., "What did I do wrong here?"; "Does the solution make sense?").

Interestingly, in almost all studies, the metacognitive instruction was employed in cooperative settings where small groups of 4–6 students studied together (e.g., Schoenfeld, 1987; Mevarech and Kramarski, 1997; Hoek, Van den Eeden and Terwel, 1999). The cooperative-metacognitive approach is based on cognitive theories of learning that emphasize the important role of elaboration in constructing new knowledge (Wittrock, 1989), and on a large body of research (e.g., Davidson, 1990; Qin, Johnson and Johnson, 1995; Stacey and Kay, 1992; Webb, 1991, 1989) showing that cooperative learning has the potential to improve mathematical achievement because it provides a natural setting for children to supply explanations and elaborate their reasoning. Since this potential has not always materialized, researchers suggested the embedding of metacognitive instruction in cooperative settings in order to provide an appropriate condition for students to elaborate their mathematical reasoning (Schoenfeld, 1992; Mevarech and Kramarski, 1997; Mevarech, 1999; Kramarski, 2000; Kramarski, Mevarech and Liberman, 2001).

Generally speaking, researchers (e.g., Schoenfeld, 1992; Mevarech and Kramarski, 1997; Mevarech, 1999; Hembree, 1992) reported positive effects of cooperative-metacognitive instruction on students' mathematical achievement. Kramarski and Mevarech (in press) further indicate that the effects of cooperative-metacognitive instruction were more positive than the effects of individualized-metacognitive instruction which in turn were more positive than the effects of cooperative or individualized settings with no metacognitive instruction. There is also evidence showing that the effects of metacognitive instruction employed in mathematics and foreign language classrooms on mathematics achievement were more positive than the effects of metacognitive instruction employed only in mathematics classrooms which in turn were more positive than no-metacognitive instruction (Kramarski, Liberman and Mevarech, 2001). The contribution

of metacognitive instruction has been documented not only with regard to mathematical problem solving, but also with regard to reading comprehension (Cross and Paris, 1988; King, 1989, 1991, 1994; Salomon, Globerson, and Guterman, 1989). Cross and Paris (1988), for example, emphasize the effectiveness of reflecting on reading strategies, King (1989, 1991, 1994) enhances the bridging of existing and new knowledge as a means for improving reading comprehension. Salomon, Globerson, and Guterman (1989) reported that students who were exposed to metacognitive instruction in reading classrooms improved not only their reading comprehension skills, but also their writing ability assessed as a transfer task. It seems that students who were exposed to metacognitive instruction were able to generalize the activation of metacognitive processes beyond a specific domain. Given these studies, we hypothesized that metacognitive instruction embedded in cooperative mathematics classrooms would exert positive effects not only on students' ability to solve standard problems as found in previous studies (e.g., Mevarech and Kramarski, 1997; Cardel-Elawar, 1995), but also on students' ability to solve authentic tasks.

Another open question relates to the differential effects of metacognitive instruction on lower and higher achievers. As indicated, lower and higher achievers face different difficulties in solving authentic tasks. Thus, it is not clear how lower and higher achievers who were exposed to metacognitive instruction would approach authentic tasks. Would lower achievers under metacognitive instruction be able to see the task as a whole, and refer to all the data given in the authentic tasks? Would they be able to reorganize and process the quantitative information in an effective way? Would they be able to draw conclusions based on their mathematical reasoning? Similarly, how would higher achievers who were exposed to metacognitive instruction approach novel, authentic tasks for which there are no ready-made algorithms? Would they be able to bridge between what they know about standard tasks and authentic tasks? The present study addresses these issues.

Finally, even with regard to standard tasks, many questions are still open, particularly those that relate to differential effects of metacognitive instruction on lower and higher achievers. Reviewing the literature indicates that previous findings were inconsistent. While some studies showed that lower achievers benefited from metacognitive instruction more than higher achievers (e.g., Cardelle-Elawar, 1995), others reported the reverse phenomenon (e.g., Hoek, et al., 1999), and still others did not find statistical interactions between prior knowledge and the treatment (e.g., Mevarech and Kramarski, 1997; Mevarech, 1999; Fuchs, Fuchs, Hamlett and Karns, 1998). One factor that may explain the inconsistency is the dif-

ferent experience that teachers had in implementing the metacognitive instruction, which in turn might affect differently their students' achievement. Thus, it would be interesting to examine the differential effects of metacognitive instruction when implemented by teachers who had much experience with the method.

Since the solution of authentic tasks requires the integration of various sources of knowledge and skills, and since more of the studies reviewed above focused on authentic tasks it is interesting to examine the differential effects of metacognitive instruction on the way lower and higher achievers approach the various aspects of authentic tasks. The purpose of the present study is, therefore, threefold: (a) to compare the effects of cooperative-metacognitive (COOP+META) and cooperative instruction (COOP) on students' ability to solve mathematical authentic tasks; (b) to examine the differential effects of these two methods on lower and higher achievers; and (c) to replicate the findings of previous studies regarding the effects of cooperative-metacognitive and cooperative instruction on students' solutions of standard tasks, with teachers who use metacognitive instruction regularly in their mathematics classrooms. The latter is important because it may show the lasting effects of metacognitive instruction beyond the experimental period which is often affected by the Hawthorn effect and in which teachers often receive assistance from mentors, researchers, or other experts.

METHOD

Participants

Participants were 91 seventh-grade students (44 boys and 47 girls) who studied in three heterogeneous classrooms selected from two junior high schools. Each school is an integrated school composed of students from different socioeconomic backgrounds as defined by the Israel Ministry of Education. Classes were similar in terms of size, students' mean age (mean age 12.3 years old), and levels of mathematics achievement assessed prior to the beginning of the study (see below).

Treatment

In the present study, students studied mathematics under one of two conditions: cooperative learning embedded within metacognitive instruction (COOP+META) and cooperative learning with no metacognitive instruction (COOP). The two conditions were identical in terms of: the lesson structure, the mathematical tasks the students solved, and the number of

hours mathematics was taught (5 periods a week). Below is a detailed description of each component.

Under both conditions each period included three parts: a) teacher's introduction to the whole class (about 10 minutes); (b) practicing activities in small heterogeneous groups (about 30 minutes); and (c) teachers' review of the main ideas of the lesson with the whole class (about 5 minutes). When common difficulties were observed, the teacher provided additional explanations.

The small heterogeneous groups were composed of four students: one high achiever, one low achiever, and two middle achievers. Students were assigned to groups by the teacher according to her ordinary evaluation.

In all three classrooms, students were exposed to the same tasks. All tasks were standard. None of the students was introduced to authentic tasks prior to the beginning of the study or during the ongoing instruction.

The differences between the treatments were only with regard to the metacognitive instruction, as follows:

The COOP+META condition: We used the **IMPROVE** method (Mevarech and Kramarski, 1997; Mevarech, 1999; Kramarski, Liberman, and Mevarech, 2001) to train students to activate metacognitive processes in small groups. Students were taught to formulate and answer four kinds of self-addressed metacognitive questions: *comprehension*, *connection*, *strategic*, and *reflection questions*.

The *comprehension questions* were designed to prompt students to reflect on the problem/task before solving it. In addressing a comprehension question, students had to read the problem/task aloud, describe the task in their own words, and try to understand what the task/concept means. The *comprehension questions* included questions such as: "What is the problem/task all about?"; "What is the question?"; "What are the meanings of the mathematical concepts?"

The *connection questions* were designed to prompt students to focus on similarities and differences between the problem/task they work on and the problem/task or set of problems/tasks that they had already solved. For example: "How is this problem/task different from/similar to what you have already solved? Explain why."

The *strategic questions* were designed to prompt students to consider which strategies are appropriate for solving the given problem/task and for what reasons. In addressing the strategic questions, students had to describe the

what (e.g., "What strategy/tactic/ principle can be used in order to solve the problem/task?") the *why* (e.g., "Why is this strategy/tactic/principle most appropriate for solving the problem/task?") and *how* (e.g., "How can I organize the information to solve the problem/task?"; and "How can the suggested plan be carried out?").

The *reflection questions* were designed to prompt students to *reflect* on their understanding and feelings during the solution process (e.g., "What am I doing?"; "Does it make sense?"; "What difficulties/feelings do I face in solving the task?"; "How can I verify the solution?"; "Can I use another approach for solving the task?").

The teachers modeled the use of the metacognitive questions in their introductions, reviews and when they provided help in the small groups. Following the teacher's introduction, students started to work in small groups using the materials designed by us.

The studying in the small groups was implemented as follows: each student, in his/her turn, read the task aloud and tried to solve it and explain his or her mathematical reasoning. Whenever there was no consensus, the group discussed the issue until the disagreement was resolved. Students were encouraged to talk about the task, explain it to each other, and approach it from different perspectives. Students used the metacognitive questions during their discourse in small group activities and in their written explanations when they solved the mathematical tasks. When all students agreed upon the solution, they wrote it down in their notebooks.

The COOP condition: Under this condition, students studied in small heterogeneous groups, but they did not use the metacognitive questions. Each student in his or her turn read the task aloud, tried to solve it and explain his or her reasoning. When he or she failed to solve the task or when students did not agree on the solution, the team discussed the task until a consensus was achieved. When all team members agreed on a solution, they wrote it down in their notebooks. When none of the team members knew how to solve the task, they asked for teacher help. As indicated, the COOP students were exposed to the same tasks as the COOP+META students.

Teacher training and learning materials

Three teachers participated in the present study. Each taught one classroom. All teachers were female and had a similar level of education (B. Ed majoring in mathematics), and more than five years of experience in teaching mathematics in heterogeneous classrooms.

Prior to the beginning of the study, all three teachers were exposed to a one-day in-service training. They were told that they will participate in a study in which new materials on problem solving are being tried out. They went over the new materials and learned how to use them with their students. The materials included the tasks and the explicit lesson plans. In addition, the teachers were introduced to pedagogical issues related to the New Standards (NCTM, 2000). A discussion was held about the question: "What is a worthwhile mathematical problem?"

The teachers who were assigned to the IMPROVE method were familiar with it. They started to implement the method three years prior to the beginning of the study, and since then they have used the method regularly in their mathematics classrooms. Nevertheless, prior to the beginning of the study, we briefly reviewed the IMPROVE principles and reminded the teachers to embed the metacognitive questions in the new unit. The teacher of the COOP group implemented cooperative learning regularly in her mathematics classrooms. She was not introduced to the metacognitive technique.

Measurements

Two measurements were used in the present study, one for assessing prior mathematical knowledge, and the other for the post-testing. Each measurement was composed of two parts: authentic and standard tasks. For the sake of simplicity, all scores on the standard and authentic tasks have been presented in terms of percent correct answers. It should be noted that in no classroom did the students study the solution of authentic tasks.

Mathematics prior knowledge examination

Authentic task

The *authentic task* entails arranging a hall for a party in school (see below). Students received three different price proposals for renting the hall. The prices varied as a function of the number of participants in the party. Each student had to decide which offer is most worthwhile in his or her opinion, and justify his/her proposal in using various representations.

The hall task. You have to arrange a hall for a party in school. Here are three different price proposals for renting the hall:

- The price is NIS 1,000 for renting the hall, no matter how many people will arrive.
- The basic price for renting the hall is NIS 400. But, if the number of participants will be more than 200, you have to pay NIS 3 more for each person.

- The basic price for renting the hall is NIS 200 and in addition you have to pay NIS 3 for each participant.

Decide which offer is most worthwhile. Explain your reasoning.

Scoring. The hall task scores ranged from 0 (no response or incorrect answer) to 3 (full correct answer). A full correct answer regards organizing correctly the information by using tables, diagrams or algebraic expressions, making a correct suggestion based on the given information and providing verbal explanations to justify the suggestion.

Standard tasks

The *standard tasks* was designed by Mevarech and Kramarski (1997). It included 41 tasks that covered the following topics, all taught prior to the beginning of the study: whole numbers, fractions, decimals and percents. The standard tasks were based on multiple-choice items regarding basic factual knowledge and open-ended computation tasks.

Scoring. For each item, students received a score of either 1 (correct answer) or 0 (no response or incorrect answer), and a total score ranging from 0 to 41. Kuder Richardson reliability coefficient was $\alpha = .87$.

Mathematics posttest

The authentic task

We used the Pizza task described shortly above to assess students' ability to solve an authentic task.

The pizza task. Your classmates organize a party. The school will provide the soft drinks, and you are asked to order the pizzas. The class budget is NIS 85.00. Of course, you want to order as many pizzas as you can. Here are proposals of three local pizza restaurants and their prices. Compare the prices and suggest the cheapest offer to the class treasurer. Write a report to the class treasurer in which you justify your suggestion.

TYPE OF PIZZA	PRICE PER PIZZA	DIAMETER	PRICE FOR SUPPLEMENTS
<i>PIZZA BOOM</i>			
PERSONAL PIZZA	3.50 NIS	15	4.00 NIS
SMALL	6.50 NIS	23	7.75 NIS
MEDIUM	9.50 NIS	30	11.00 NIS
LARGE	12.50 NIS	38	14.45 NIS
EXTRA LARGE	15.50 NIS	45	17.75 NIS
<i>SUPER PIZZA</i>			
SMALL	8.65 NIS	30	9.95 NIS
MEDIUM	9.65 NIS	35	10.95 NIS
LARGE	11.65 NIS	40	12.95 NIS
<i>MC PIZZA</i>			
SMALL	6.95 NIS	25	1.00 NIS
LARGE	9.95 NIS	35	1.25 NIS

The Pizza Task has all the characteristics of authentic tasks described above. The situation is most familiar to junior high school students, the mathematical data is rich, and there is no ready-made algorithm for solving it. Quite often children at this age go to restaurants that offer pizzas having different prices, sizes, and supplements; and quite often they have to decide what kind of pizza to choose. The Pizza Task requires the use of a variety of sources of information (e.g., prices, size, number of supplements) and it has many different correct solutions. The solvers have to make computations, use different representations, and apply knowledge regarding geometry, fractions and ratio.

Scoring. Students' responses were scored on four criteria: (a) Referring to all data; (b) Organizing information; (c) Processing information; and (d) Making an offer and justifying it. Each criterion was scored between 0 (no response or incorrect response) to 5 (full correct response). Appendix 1 provides examples of the scoring.

A full correct answer regards the organizing of the information in a table, diagram, or an algebraic expression, making a correct suggestion based on the given information, and justifying the suggestion by explaining one's mathematical reasoning.

The following is a detailed description of each criterion:

Referring to all data – referring to all the relevant data in each of the three proposals: price per pizza, price for supplements, and pizza diameter.

Organizing information – Arranging the data in a table, diagram, algebraic expressions or any other representation.

Processing information – Making the calculations correctly, and describing explicitly the solution process.

Making an offer and justifying it – Presenting a proper solution to the required task, reporting the suggestions in a letter to the treasurer, giving reasons to justify one's choice, and maximizing exploitation of the money. Inter-judge reliability of four categories was .90.

Standard tasks

This part included 22 standard tasks that covered the following topics: rational numbers, identification of rational numbers on the number axis, operations with positive and negative numbers, order of operations, and the use of algebraic expressions. The test was composed of multiple-choice items regarding basic factual knowledge, and open-ended computation tasks. The following is an example of a standard task.

Osnat has 4 times more disks than Hani has. If x represents Hani's number of disks, write an algebraic expression that represents the number of disks that Osnat has; If y represents Osnat's number of disks, write an algebraic expression that represents the number of disks that Hani has.

Scoring. For each item, students received a score of either 1 (correct answer) or 0 (no response or incorrect answer), and a total score ranging from 0 to 22. To simplify the report all scores are presented in terms of percent correct answers. Kuder-Richardson reliability coefficient was $\alpha = .86$.

Procedure

At the beginning of the school year, all students were administered the pretest. After the pre-testing, each teacher started teaching according to the instructional method to which she was assigned using the materials specially designed for that treatment. Students continued to study mathematics in cooperative settings with or without metacognitive training throughout the whole year. Students were assigned to small groups by the teachers according to her ordinary evaluation. The present study focused, however, only on the problem solving unit which was taught in all classrooms in the last semester for six weeks. During these six weeks, observers observed each class twice a week to ensure that each teacher implemented the method as designed. At the end of the study, the post-test was administered to all students.

TABLE I

Mean scores and standard deviations on the authentic tasks, by treatment and prior knowledge (scores are reported in terms of percent correct answers)

	COOP		COOP+META	
	Lower achievers N = 14	Higher achievers N = 17	Lower achievers N = 31	Higher achievers N = 29
Pretest				
Authentic hall task				
M	41.02	51.85	46.24	55.17
SD	14.60	23.49	16.50	16.20
Posttest				
Authentic pizza task (Total)				
M	35.77	46.11	44.03	69.31
SD	19.98	23.11	17.19	24.59
Reference to all data				
M	38.46	51.12	47.10	73.81
SD	22.31	22.91	21.63	23.36
Organizing information				
M	26.15	33.23	34.81	56.51
SD	17.21	28.30	12.60	34.62
Processing information				
M	30.76	41.11	43.20	66.21
SD	22.53	31.03	18.71	29.30
Making an offer and justifying it				
M	47.71	58.81	50.96	80.71
SD	28.91	25.20	26.71	26.40

RESULTS

Authentic tasks

Table I presents the mean scores and standard deviations on the pretest and posttest authentic tasks by treatment and prior knowledge. A two-way Analysis of Variance (ANOVA) was performed on the authentic pretest. One factor was the treatment (COOP, COOP+META), and the other—prior knowledge (above or below the median). According to Table I prior to the beginning of the study, neither the treatment main effect ($F(1,87) = 0.91, p > 0.05; M = 50.55; 47.31; SD = 16.79; 20.68$, for the COOP+META

TABLE II

Mean scores and standard deviations on standard tasks, by treatment and prior knowledge (scores are reported in terms of percent correct answers)

	COOP		COOP+META	
	Lower achievers N = 14	Higher achievers N = 17	Lower achievers N = 31	Higher achievers N = 29
Pretest standard tasks				
M	54.14	88.53	51.84	84.07
SD	11.73	8.86	13.29	7.26
Posttest standard tasks				
M	41.16	71.67	64.10	86.31
SD	11.32	13.86	16.58	9.11

and COOP groups respectively) nor the interaction between the treatment and prior knowledge were significant ($F(1,87) = 0.01, p > 0.05$). The only significant main effect was prior knowledge ($F(1,87) = 4.67, p < 0.03$).

Regarding the posttest a two-way (Treatment $\times$ Prior Knowledge) MANOVA was performed on the four criteria of the Pizza Task simultaneously. Results indicated significant main effects for the treatment ($F(4,84) = 3.21, p < 0.05$) and prior knowledge ($F(4,84) = 5.01, p < 0.01$), but no significant interaction was found between the treatment and prior knowledge ($F(4,84) = 0.66, p > 0.05$).

Effect-Sizes were calculated according to Cohen's formula as the difference between the mean scores of the experimental (COOP + META) and the control (COOP) groups, divided by the standard deviations of the control group. According to Table I, at the end of the study, both lower and higher achievers benefited from the metacognitive training on the three criteria: *reference to all data* (Effect-Size = .39; 1.00, for lower and higher achieving students, respectively), *organizing information* (Effect-Size = .50; .82, for lower and higher achieving students, respectively), and *processing information* (Effect-Size = .55; .81, for lower and higher achieving students, respectively). On the last criterion: *making an offer and justifying it*, significant differences were found for higher achievers but not for lower achievers (Effect-Size = .11; .87, for lower and higher achieving students, respectively).

Standard tasks

To address the question of the differential effects of metacognitive instruction on lower and higher achievers' solutions of standard tasks we analyzed the data by using a 2×2 Analysis of Variance (ANOVA). One factor was the treatment (COOP, COOP+ META), and the other prior knowledge (above or below the median). Table II reports the means scores and standard deviations of the standard tasks by treatment and prior knowledge.

Results indicated that prior to the beginning of the study no significant differences were found between the two treatments on standard tasks ($F(1,87) = .09, p > 0.05$). Also the interaction between the treatment and prior knowledge was not significant ($F(1,87) = 0.71, p > 0.05$). Only the prior knowledge main effect was significant ($F(1,87) = 199.63, p < 0.001$).

At the end of the study, however, the main effects of the treatment and prior knowledge were significant ($F(1,87) = 40.12, p < 0.0001$, and ($F(1,87) = 78.96, p < 0.0001$, for the treatment and prior knowledge, respectively). In addition, the interaction between the treatment and prior knowledge was not significant ($F(1,87) = 1.96, p < 0.1$). According to Table II, lower and higher achievers in the COOP+ META group outperformed their counterparts in the COOP group (Effect-Size = 1.93 and 1.06, respectively for lower and higher achievers), but the effect size was much higher for the lower achieving students.

DISCUSSION

The present study investigated the effects of cooperative-metacognitive instruction (COOP+ META) and cooperative instruction (COOP) on students' solutions of authentic and standard tasks. The findings indicated that the COOP+ META students significantly outperformed the COOP students on both kinds of tasks (authentic and standard). Yet, the generalizations of the findings might be limited because of at least three reasons. First, the sample was rather small involving only 91 students and three teachers. Second, only two authentic tasks were used, one for the pretest and one for the posttest. Finally, data analyses were based on paper-and-pencil measurements with no observations and/or interviews. Future research may continue to investigate the effects of metacognitive instruction on authentic tasks by utilizing a larger sample, various types of authentic tasks, and different kinds of measurements.

Supposing the findings are generalized, the study raises two main questions: (a) What is the role of metacognitive instruction in enhancing the different aspects of authentic task solutions? and (b) Why did the lower

achievers in the COOP+ META condition outperform their counterparts in the COOP condition on three criteria: *reference to all data; organizing information and processing information*, but not on *making an offer and justifying it*?

THE ROLE OF METACOGNITIVE INSTRUCTION IN ENHANCING THE SOLUTION OF AUTHENTIC TASKS

According to Lester (1994) “good problem solvers know more than poor problem solvers and their knowledge is well connected and composed of rich schemata. Good problem solvers tend to focus their attention on structural features of the problems, poor problem solvers on surface features. Good problem solvers are better than poor problem solvers at monitoring and regulating their problem solving efforts.” (p. 665). According to Hegarty, Mayer and Monk (1995) good problem solvers use a different approach than poor problem solvers: Whereas good problem solvers construct a model of the given problem on the basis of all the information given in the problem text, poor solvers translate the key words given in the problem text directly into the mathematical operations that the key words usually prime without considering other information given in the text.

Being trained to reflect on their problem solving processing probably led students to focus on the structural features of the task as well as on all the information given in the task. The metacognitive self-addressed *comprehension questions* (e.g., “what is the problem all about?”) probably guided students to look for all the relevant information, distinguish between the relevant and irrelevant information and comprehend the entire task rather than parts of it. Also the *connection questions* (e.g., “How is this problem/ task different/similar from what you have already solved?”) might lead students to pay attention to all the information and the structure of the given task. Therefore, there is reason to suppose that being trained to use the self-addressed questions resulted in students’ references to all the variables and data given in the task. Support for this hypothesis comes from the fact that the mean score of the COOP+ META students on “reference to all data” was higher than that of COOP students ($M = 60.01$ and 45.02 ; $SD = 26.04$ and 23.20 for COOP +META and COOP students respectively). Appendix 2 provides an example of how a COOP+ META student referred to all the information given in the text by offering a combination of all kinds of pizzas. In contrast, none of the COOP students suggested a similar offer. Instead, the COOP students referred partially to one kind of pizza or compared one kind of pizza to another (e.g., “I prefer the Pizza Boom because we have 24 children in the class, so I can give exactly each

child one personal pizza. $3.5 \times 24 = 84$ "). This example shows also that the COOP students used rather simple strategies based only on multiplication of the numbers, whereas the COOP+ META students carried out multiple operations.

The findings further showed that the metacognitive students were also better able to reorganize and process given information than their counterparts in the non-metacognitive condition. This is probably due to the fact that the metacognitive instruction trained students to think which strategies are appropriate for solving the task and why. By doing so, students suggested different kinds of representations, compared the strategies, and analyzed the pros and cons of each strategy.

Finally, the findings indicated that the COOP+ META students were better able to justify their reasoning than their counterparts in the cooperative condition who were also encouraged to discuss their mathematical ideas and be involved in the mathematical discourse that took place in the small group. This finding indicates that studying in cooperative settings is not sufficient for enhancing mathematical discourse, and that students' interaction in the small group has to be structured in order to ensure that all students would indeed be involved in justifying their reasoning. This finding is in line with the studies of Symons and Green (1993) and Webb (1991) who showed that asking students to answer *why* questions during the solution processes of standard problems helped them elaborate and retain information. As students explain and justify their thinking, and as they challenge the explanations of their peers and teachers, they are also engaging in clarifying their own thinking and becoming owners of "knowing" (Lampert, 1990). The *comprehension* questions, the *connection* questions, the *strategic* questions and the *reflection* questions all guided students to reflect on their own and their peers' solutions and solution processes. Without using these metacognitive questions the mathematical discourse was rather dull.

The use of *connection* questions needs further consideration. Since the metacognitive instruction was employed during the studying of standard tasks, whereas the effects of the methods were assessed on authentic tasks to which students were exposed only in the testing periods, it is possible that the metacognitive connection questions do not only give cognitive tools to students for solving complex tasks, but also strengthens students' self-confidence in approaching new kinds of tasks. An indirect support for this hypothesis comes from the fact that 65% of the cooperative-metacognitive (COOP+ META) students compared to only 38.7% of the cooperative (COOP) students approached the authentic tasks. Thus, using the metacognitive questions, and in particular the connection questions,

probably gave students the confidence to try and cope with the authentic tasks.

Another claim suggested for explaining the findings is that the positive effects of the metacognitive instruction were due to the fact that the COOP+ META participating teachers felt more engaged in the experiment. This claim should be rejected for two reasons. First, all teachers were told that they participate in a study that tries out new materials on problem solving. Second, all teachers implemented the methods regularly in their mathematics classrooms and therefore there is no reason to suppose that they were affected by Hawthorn effects. Furthermore, the very fact that these teachers used the methods regularly in their mathematics classrooms may point to the lasting effects of the methods beyond the experimental period during which teachers often receive assistance from mentors, researchers or other experts.

Although we do not have "hard data" on the "teaching quality" of the participating teachers, the fact that no significant differences were found between the conditions prior to the beginning of the study, and the fact that the teachers have the same level of education, similar years of experience, and three years of experience in implementing the specific method they used in the present study, allows us to think that the measured differences were due to the designed teaching program.

THE DIFFERENTIAL EFFECTS OF METACOGNITIVE INSTRUCTION ON LOWER AND HIGHER ACHIEVERS

Although both lower and higher achievers benefited from the metacognitive instruction, refine analyses indicated different effect-sizes for lower and higher achievers. Whereas effect-sizes of lower achievers were significant on: *reference to all data; organizing information and processing information*, the Effect-Size on *making an offer and justifying it* was not significant. Also with regard to standard tasks, the effect size shows much larger for lower than higher achievers.

Why did not lower achievers in the cooperative-metacognitive (COOP+ META) condition perform significantly better on drawing a conclusion and justifying it than their counterparts in the cooperative setting? It is possible that many of the lower achievers in mathematics have also difficulties in expressing their ideas in writing, and thus could not clearly justify their reasoning. It is also possible that the metacognitive instruction was not intensive enough to lead lower achievers to make conclusions based on all the information given in the task and on the computations they performed. Since lower achievers in the cooperative-metacognitive condition

(COOP+ META) performed significantly better on standard tasks than students in the cooperative condition, it is possible that COOP+ META lower achievers had the knowledge but could not integrate it in order to draw a conclusion and justify it. Yet, as indicated above, the small number of participants does not allow us to generalize the findings. Future research may address this issue.

The finding regarding the effects of COOP+ META on lower and higher achievers' solutions of standard tasks extends previous findings (Cardelle-Elawar, 1995; Fuchs, Fuchs, Hamlett and Karns, 1998; Mevarech, 1999; Mevarech and Kramarski, 1997). In all these studies lower achievers who learned to activate metacognitive processes were better able to solve standard problems than students who were not exposed to metacognitive instruction.

Implications and future research

The findings of the present study raise several questions for further research. First – no formal observations were conducted in this study. Thus, the quality of group interactions under the different conditions is not known. It would be particularly interesting to examine how lower and higher achievers interact with each other under the different cooperative conditions.

Second, the present study investigated the differential effects of the treatments on lower and higher achievers. The effects of the methods on middle achieving students is not known at present. Quite often middle achievers "fall between the chairs": they do not get the same attention as lower achievers, and they do not have the knowledge and skills as higher achievers. Thus, it would be interesting to investigate the differential effects of the methods on middle achieving students. Unfortunately, the small number of participants does not allow us to divide the sample into lower, middle, and higher achievers. This issue merits future research.

Finally, these findings also call for the design of additional learning environments based on similar components. The extent to which the metacognitive questions used in the present study are appropriate also for children at different grades or for different mathematical topics is not known at present and may be investigated in future research.

Assuming that these findings generalize to other settings, the study suggests several important practical implications. The study shows that there is a need to structure learning in small groups, and that features of discourse such as giving reasons must be practiced and reinforced (Webb, 1991; Webb and Farivar, 1994; Cohen, 1996; Mevarech and Kramarski, 1997).

Furthermore, the norms for enhancing mathematical authentic tasks in high-school classrooms are not well developed. One of the challenges that mathematical education faces is to select and develop “worthwhile problems” for the lower and higher achievers. “Regardless of the context, worthwhile tasks should be intriguing, with a level of challenges that invites speculation and hard work” (NCTM, 2000, p. 19). Such problems can promote students’ conceptual understanding, foster their ability to reason and communicate mathematically, and capture students’ interest and curiosity (NCTM, 2000).

At present, many state proficiency tests and international examinations (e.g., TIMSS-1999 or PISA-2000 administered by OECD-2000 countries) include authentic tasks that ask students to explain their reasoning in writing. To acquaint students with such tasks, teachers may prepare guidelines and ask students to score one another’s reasoning by using the guidelines and activating metacognitive processes. Furthermore, researchers in the area of mathematics education may design “item banks” that include authentic tasks that challenge students to provide explanations.

APPENDIX 1:

Examples for scoring the pizza task.

Example 1:

I think that we have to buy a large pizza at the “SUPER PIZZA”. We can buy 7 super pizzas and it will cost NIS 81.55. I will divide every pizza into 8 pieces and it will be enough for 56 children. We will not order supplements. We will be left with NIS 3.45 I think that we should buy that pizza.

Scoring:

1. *Reference to all data:* The student referred to only one of the proposals – score 1.
2. *Organizing information:* The student did not use any representation to present his calculation or conclusion – score 0.
3. *Processing information:* The calculations are correct, but the student did not write explicitly the solution process – score 3.
4. *Making an offer and justifying it:* The student wrote his choice (“we have to buy a large pizza”), but he did not justify it nor did he compare his conclusion to other possibilities (e.g., “we will not order supplements”) – score 3.

Total score: 7

Example 2:

PIZZA BOOM				
Type of pizza ¹	Number of pizzas ²	Total number of personal pieces ³	Price per pizza ⁴	Price to pay ⁵
Personal pizza	85:3.5~24	24	3.5	84
Small: 4 pieces per pizza	85:6.5~13	$13 * 4 = 52$	6.5	84.5
Medium: 6 pieces per pizza	85:9.5~9	$9 * 6 = 54$	9.5	85.5
Large: 8 pieces per pizza	85:12.5~7	$7 * 8 = 56$	12.5	87.5
X-Large: 10 pieces per pizza	85:15.5~5	$5 * 10 = 50$	16.5	77.5

^a I calculated the number of pizzas² I could buy for NIS 85 and then multiplied the results with the number of pieces we have in each pizza¹ and I got the total number of personal pieces³.

Note^a: The student explained how he calculated, approximately the number of pizzas² he could buy for NIS 85 and how he got the number of personal pieces³. We can also see from the table that he multiplied the number of pizzas² by price per pizza⁴ and got the total price to pay⁵.

SUPER PIZZA				
Type of pizza	Number of pizzas	Total number of personal pieces	Price per pizza	Price to pay
4 small	85:8.65~10	$10 * 4 = 40$	8.65	86.5
6 medium	85:9.65~9	$9 * 6 = 54$	9.65	86.85
8 large	85:11.65~7	$7 * 8 = 56$	11.65	81.55

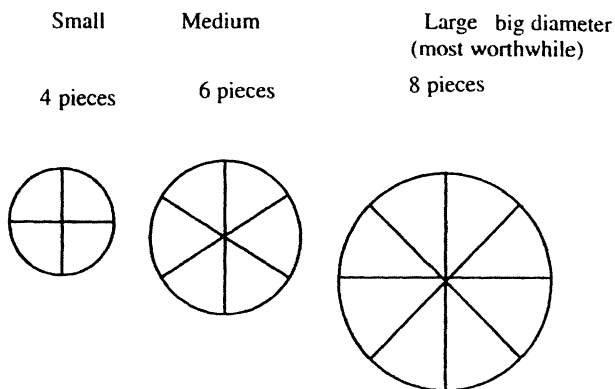
MC PIZZA				
Type of pizza	Number of pizzas	Total number of personal pieces	Price per pizza	Price to pay
4 small	85:6.95~12	$12 * 4 = 48$	6.95	83.4
8 large	85:9.95~8	$8 * 8 = 64$	9.95	79.6

Dear Class Treasurer,

The results of my calculations:

The best pizza for us to buy is from MC Pizza. We can buy 8 large pizzas (it means big diameter), and in each one 8 pieces. We get 64 pieces and have to pay only NIS 79.6

There is money left for a drink. It is great !!!!!



Scoring:

1. *Reference to all data:* The student referred to the class budget (NIS 85) and to the data of the three proposals. But he did not refer to the diameter of the pizza and the price for supplements – score 3.
2. *Organizing information:* The student used various representations: table, diagrams of pie and verbal explanations – score 4.
3. *Processing information:* The calculations are correct and the student wrote clearly the solution process – score 5.
4. *Making an offer and justifying it:* The student explained his choice and justified his reasoning – score 5.

Total score: 17

APPENDIX 2:

Example of a mixed offer based on a combination of all types of the pizza's prices and supplements (given by a student under the metacognitive condition).

PIZZA BOOM –

Medium –	9.5
Large –	12.5
Extra Large –	15.5
<i>Total –</i>	<i>37.5</i>

SUPPER PIZZA –

Medium –	9.65
Large –	11.65
Small –	9.95
<i>Total –</i>	<i>32.15</i>

MC PIZZA –

Large –	9.95
Personal from Pizza Boom –	3.5
Supplement for 2 small Pizzas –	2
<i>Total –</i>	<i>15.45</i>

$$37.5 + 32.15 + 15.45 = 85.1$$

REFERENCES

- Anderson, J.R.: 1990, *Cognitive Psychology and Its Implications*, (3rd ed.), Freeman, New York.
- Brown, A.L., Bransford, J.D. Ferrara, R.A. and Campione, J.C.: 1983, 'Learning, remembering, and understanding', in J.H. Flavell and E.M. Markman (eds.), *Handbook of Child Psychology: Vol. 3, Cognitive Development*, Wiley, New York, pp. 77–166.
- Cai, J.: 2000, *Problem-Based Mathematics Instruction: Promises and Challenges*, Paper presented at the ICME-9 Congress in Makuhari, Japan, in Topic Group 11, Problem Solving in Mathematics.
- Cardelle-Elawar, M.: 1995, 'Effects of metacognitive instruction on low achievers in mathematics problems', *Teaching and Teacher Education* 11(1), 81–95.
- Cobb, P.: 1994, 'Where is the mind? Constructivist and sociocultural perspectives on mathematical development', *Educational Researcher* 23, 13–20.

- Cohen, E.G.: 1996, *A Sociologist Looks at Talking and Working Together in the Mathematics Classroom*, Paper presented at the American Educational Research Association, NY.
- Cross, D.R. and Paris, S.C.: 1988, 'Developmental and instrumental analysis of children's metacognition and reading comprehension', *Journal of Educational Psychology* 80, 131–142.
- Darling-Hammond, L.: 1992, *Reforming the School Reform Agenda: Developing Capacity for School Transformation*, Eric ED. 347656.
- Davidson, N.: 1990, *Cooperative Learning in Mathematics*, Academic Press, New York.
- Flavell, J.H.: 1978, 'Metacognitive development', in J.M. Scandura and C.J. Brainerd (eds.), *Structural Process Theories of Complex Human Behavior*, Sijtoff & Noordhoff, Alphen a/d Rijn, The Netherlands, pp. 213–245.
- Forman, S.L. and Steen, L.A.: 2000, *Making Authentic Mathematics Work for all Students*, Eric ED. 440848.
- Fry, S.M.: 1987, 'The NCTM standards: Challenges for all classrooms', *Mathematics Teacher* 2, 313–317.
- Fuchs, L.S., Fuchs, D., Hamlett, C.L. and Karns, K.: 1998, 'High-achieving students' interactions and performance on complex mathematical tasks as a function of homogeneous and heterogeneous pairings', *American Educational Research Journal* 35(2), 227–267.
- Hattie, J., Biggs, J. and Purdie, N.: 1996, 'Effects of learning skills interventions on students learning: a meta-analysis', *Review of Educational Research* 66, 99–136.
- Hembree, R.: 1992, 'Experiments and relational studies in problem solving: A meta-analysis', *Journal for Research in Mathematics Education* 23(3), 242–273.
- Hegarty, M., Mayer, R.E. and Monk, C.A.: 1995, 'Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers', *Journal of Educational Psychology* 87, 18–32.
- Hoek, D., Eden, P. and Terwel, J.: 1999, 'The effects of integrated social and cognitive strategy instruction on the mathematics achievement in secondary education', *Learning and Instruction* 9, 427–448.
- Jacobs, J. and Paris, S.: 1987, 'Children's metacognition, measurement, and instruction', *Educational Psychologist* 22, 255–278.
- King, A.: 1989, 'Effects of self-questioning training on college students' comprehension of lectures', *Contemporary Educational Psychology* 14, 366–381.
- King, A.: 1991, 'Effects of training in strategic questioning on children's problem-solving performance', *Journal of Educational Psychology* 83, 307–317.
- King, A.: 1994, 'Guiding knowledge construction in the classroom: Effects of teaching children how to question and how to explain', *American Educational Research Journal* 31, 338–368.
- Kramarski, B.: 2000, 'The effects of different instructional methods on the ability to communicate mathematically', in Tadao Nakahara and Masataka Koyama (eds.), *Education, Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Mathematical Reasoning*, Hiroshima University, Hiroshima, Japan, 1, pp. 167–171.
- Kramarski, B., Mevarech, Z.R. and Liberman, A.: 2001, 'The effects of multilevel – versus unilevel-metacognitive training on mathematical reasoning', *Journal for Educational Research* 94(5), 292–300.
- Kramarski, B., and Mevarech, Z.R.: in press, 'Enhancing mathematical reasoning in the classroom; Effects of cooperative learning and metacognitive training', *American Educational Research Journal*.

- Lampert, L.: 1990, 'When the problem is not the question and the solution is not the answer: Mathematical; knowing and teaching', *American Educational Research Journal* 27(1), 29–63.
- Lester, F.K.: 1994, 'Musing about mathematical problem solving research: 1970–1994', *Journal for Research in Mathematical Education* 25(6), 660–675.
- Lester, F.K., Garofalo, J. and Kroll, D.L.: 1989, *The Role of Metacognition in Mathematical Problem Solving: A Study of Two Grade Seven Classes*, (Final Report), Indiana University, Mathematics Education Development Center, Bloomington.
- Mayer, E.R.: 1987, *Educational Psychology*, Little Brown, Boston.
- Mevarech, Z.R.: 1999, 'Effects of metacognitive training embedded in cooperative settings on mathematical problem solving', *The Journal of Educational Research* 92(4), 195–205.
- Mevarech, Z.R. and Kramarski, B.: 1997, 'IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms', *American Educational Research Journal* 34(2), 365–395.
- National Council of Teacher of Mathematics: 2000, *Principles and Standards for School Mathematics*, Author, Reston, VA.
- OECD: 2000, *Measuring Students Knowledge and Skills*, The PISA 2000 assessment of reading, mathematical and scientific literacy.
- Prawat, R.S.: 1998, 'Current self-regulation views of learning and motivation viewed through a Deweyan lens: The problems with dualism', *American Educational Research Journal* 35(2), 199–224.
- Pressley, M., Borkowski, J.G. and O'Sullivan, J.T.: 1985, 'Children metamemory and the teaching of memory strategies', in D.L. Forest-Pressley, G.E. Mackinnon and T.G. Waller (eds.), *Metacognition, cognition, and human performance*, Academic Press, New York.
- Qin, Z., Johnson, D.W. and Johnson, R.T.: 1995, 'Cooperative versus competitive efforts and problem solving', *Review of Educational Research* 65, 129–144.
- Rose, T. and Schuncke, G.M.: 1997, 'Problem solving. The link between social studies and mathematics', *Cleaning House* 70(3), 137–140.
- Salomon, G., Globerson, T., and Guterman, E.: 1989, 'The computer as a zone of proximal development: Internalizing reading-related metacognition from a reading partner', *Journal of Educational Psychology* 81, 620–627.
- Schoenfeld, A.H.: 1987, 'What's all the fuss about metacognition?' in A.H. Schoenfeld (ed.), *Cognitive Science and Mathematics Education*, Erlbaum, Hillsdale, NJ, pp. 19–216.
- Schoenfeld, A.H.: 1992, 'Learning to think mathematically: problem solving, metacognition, and sense making in mathematics', in D.A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, MacMillan, New York, pp. 165–197.
- Silver: 1994, 'Mathematics thinking and reasoning for all students: moving from rhetoric to reality', in D. Robitaille, D. Wheeler and C. Kieran (eds.), *Selected Lectures from the 7th International Congress on Mathematical Education*, Le presse de l'universite Laval, Sainte-Foy, Quebec, Canada, pp. 311–326.
- Stacey and Kay: 1992, 'Mathematical problem solving in groups – Are two heads better than one?' *Journal of Mathematical Behavior* 11(3), 261–275.
- Symons, S. and Greene, C.: 1993, 'Elaborative interrogation and children's learning of unfamiliar facts', *Applied Cognitive Psychology* 7, 219–228.
- Swanson, H.L.: 1990, 'Influence of metacognitive knowledge and aptitude on problem solving', *Journal of Educational Psychology* 82(2), 306–314.

- TIMSS: 1999, *Third International Mathematics and Science Study*, The International Study Center, Lynch School of Education, Boston College.
- Verschaffel, L., Greer, B. and De Corte, E.: 2000, *Making Sense of Word Problems*, Swets and Zeitlinger, Lisse, ISBN 90-265-1628-2.
- Webb, N.M.: 1989, 'Peer interaction and learning in small groups', *International Journal of Educational Research* 13, 21–40.
- Webb, N.M.: 1991, 'Task-related verbal interaction and mathematics learning in small groups', *Journal for Research in Mathematics Education* 22, 366–389.
- Webb, N. and Farivar, S.: 1994, 'Promoting helping behavior in cooperative small groups in middle school mathematics', *American Educational Research Journal* 31, 369–396.
- Wittrock, M.C.: 1986, 'Students' thought processes', in M.C. Wittrock (ed.), *Handbook of Research on Teaching*, (3rd ed.), Macmillan, New York.

*School of Education,
Bar-Ilan University,
52900 Ramat-Gan,
Israel*