PERTEMUAN XIII

Metode Substitusi

Teorema. Andaikan g fungsi yang dapat diintegralkan dan andaikan bahwa F adalah anti turunan dari fungsi f. Maka jika $u = g(x)$,

$$\int f(g(x))g'(x)\ dx = \int f(u)du = F(u) + C = F(g(x)) + C$$

Contoh 1.

$$\begin{aligned}\int x^3\sqrt{x^4+11}\ dx &= \frac{1}{4}\int\sqrt{x^4+11}\ \left(4x^3dx\right)\\ &= \frac{1}{4}\int\sqrt{x^4+11}\ d\left(x^4+11\right)\\ &= \frac{1}{6}\left(x^4+11\right)^{\frac{3}{2}} + C\end{aligned}$$

Contoh 2.

$$\begin{aligned}\int\frac{x}{\cos^2\left(x^2\right)}\ dx &= \frac{1}{2}\int\frac{1}{\cos^2\left(x^2\right)}2xdx\\ &= \frac{1}{2}\int\sec^2(u)du\\ &= \frac{1}{2}\tan u + C\\ &= \frac{1}{2}\tan\left(x^2\right) + C\end{aligned}$$

Integral Fungsi Trigonometri

❖ **Jenis I :** $\int \sin^n x \, dx$ **dan** $\int \cos^n x \, dx$

Utk n ganjil $\longrightarrow$ gunakan indentitas $\sin^2 x + \cos^2 x = 1$ Dgn kata lain, dijadikan fungsi berpangkat genap dahulu (dipisah)

Contoh :

$$\int \cos^5 x \, dx = \int \cos^4 x \ \cos x \, dx$$
$$= \int (\ldots\ldots\ldots\ldots\ldots\ldots\ldots)^{\cdots\cdots} d(\ldots\ldots)$$
$$= \int (\ldots\ldots\ldots\ldots\ldots\ldots\ldots) \, d(\ldots\ldots)$$
$$= \int d(\ldots\ldots) - \ldots\int \ldots\ldots\ldots \, d(\ldots\ldots) + \int \ldots\ldots\ldots \, d(\ldots\ldots)$$
$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Utk n genap $\longrightarrow$ gunakan kesamaan setengah sudut : $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$ $\sin^2 x = \frac{1 - \cos 2x}{2}$ dan $\cos^2 x = \frac{1 + \cos 2x}{2}$

Contoh :

$$\int \sin^4 8x \, dx = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

❖ **Jenis II :** $\int \sin^m x \, \cos^n x \, dx$

Utk m atau n positif ganjil $\longrightarrow$ gunakan indentitas

$$\sin^2 x + \cos^2 x = 1$$

Dgn kata lain, dijadikan fungsi berpangkat genap dahulu (dipisah)

Contoh :

$$\int \sin^5 x \, \cos^{-2} x \, dx = \int \sin^4 x \, \cos^{-2} x \, \sin x \, dx$$

$$= \ldots\ldots \int \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \, d(\ldots\ldots\ldots)$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Utk m atau n positif genap gunakan kesamaan setengah sudut :

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad \text{dan} \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

Contoh :

$$\int \sin^4 x \, \cos^2 x \, dx = \int \left(\frac{\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots} \right)^2 \left(\frac{\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots} \right) dx$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

❖ **Jenis III :** $\int \tan^n x\, dx,\ \int \cot^n x\, dx$

Gunakan indentitas $\tan^2 x = \sec^2 x - 1$ dan $\cot^2 x = \csc^2 x - 1$ Dgn kata lain, dijadikan fungsi berpangkat genap dahulu (dipisah)

Contoh :

1. $\int \tan^4 x\, dx = \int$................................. dx

2. $\int \cot^5 x\, dx = \int$................................. dx (Lanjutkan!)

❖ **Jenis IV :** $\int \tan^m x\ \sec^n x dx,\ \int \cot^m x\ \csc^n x\, dx$

Utk n genap dan m sebarang $\longrightarrow$ gunakan identitas : $\tan^2 x = \sec^2 x - 1$ dan $\cot^2 x = \csc^2 x - 1$ Terlebih dahulu dijadikan fs. berpangkat genap (dipisah)

Contoh :

$\int \tan^{-4} x\ \sec^2 x\, dx = \int$.. dx

Utk m ganjil dan n sebarang $\longrightarrow$ gunakan identitas : $\tan^2 x = \sec^2 x - 1$ dan $\cot^2 x = \csc^2 x - 1$ Terlebih dahulu dijadikan fs. berpangkat genap (dipisah)

Contoh :

$\int \tan^5 x\ \sec^{-4} x\, dx = \int$.. dx (Lanjutkan!)

❖ **Jenis V :**

$$\int \sin mx \cos nx \, dx = \int \tfrac{1}{2}\left[\sin(m+n)x + \sin(m-n)x\right] dx$$
$$\int \sin mx \sin nx \, dx = \int -\tfrac{1}{2}\left[\cos(m+n)x - \cos(m-n)x\right] dx$$
$$\int \cos mx \cos nx \, dx = \int \tfrac{1}{2}\left[\cos(m+n)x + \cos(m-n)x\right] dx$$

Contoh :

$\int \sin 4y \ \cos 5y \ dy = \int$ .... dx (Lanjutkan!)