Tentukan $M(x,y)$ sehingga PD berikut merupakan PD Eksak.

$$M(x,y)\,dx + (2x^2y^3 + x^4y)\,dy = 0$$

→ Syarat :

$$M(x,y) = \frac{\partial F}{\partial x}(x,y) \quad \text{dan} \quad N(x,y) = \frac{\partial F}{\partial y}(x,y)$$

$$\frac{\partial M}{\partial y} = \frac{\partial^2 F}{\partial y \partial x} \quad \text{dan} \quad \frac{\partial N}{\partial x} = \frac{\partial^2 F}{\partial y \partial x}$$

sehingga

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Jawab :

$$M(x,y)\,dx + (2x^2y^3 + x^4y)\,dy = 0$$

$$\frac{\partial M}{\partial y} = M'(x,y) \qquad \frac{\partial N}{\partial x} = 4xy^3 + 4x^3y$$

$$M'(x,y) = 4xy^3 + 4x^3y$$

$$M(x,y) = \int 4xy^3 + 4x^3y\,dy$$

$$M(x,y) = xy^4 + 2x^3y^2$$

Jadi, nilai $M(x,y)$ yang berlaku, yaitu $xy^4 + 2x^3y^2$, sehingga persamaan diferensial yang memenuhi PD Eksak, yaitu

$$(xy^4 + 2x^3y^2)\,dx + (2x^2y^3 + x^4y)\,dy = 0$$