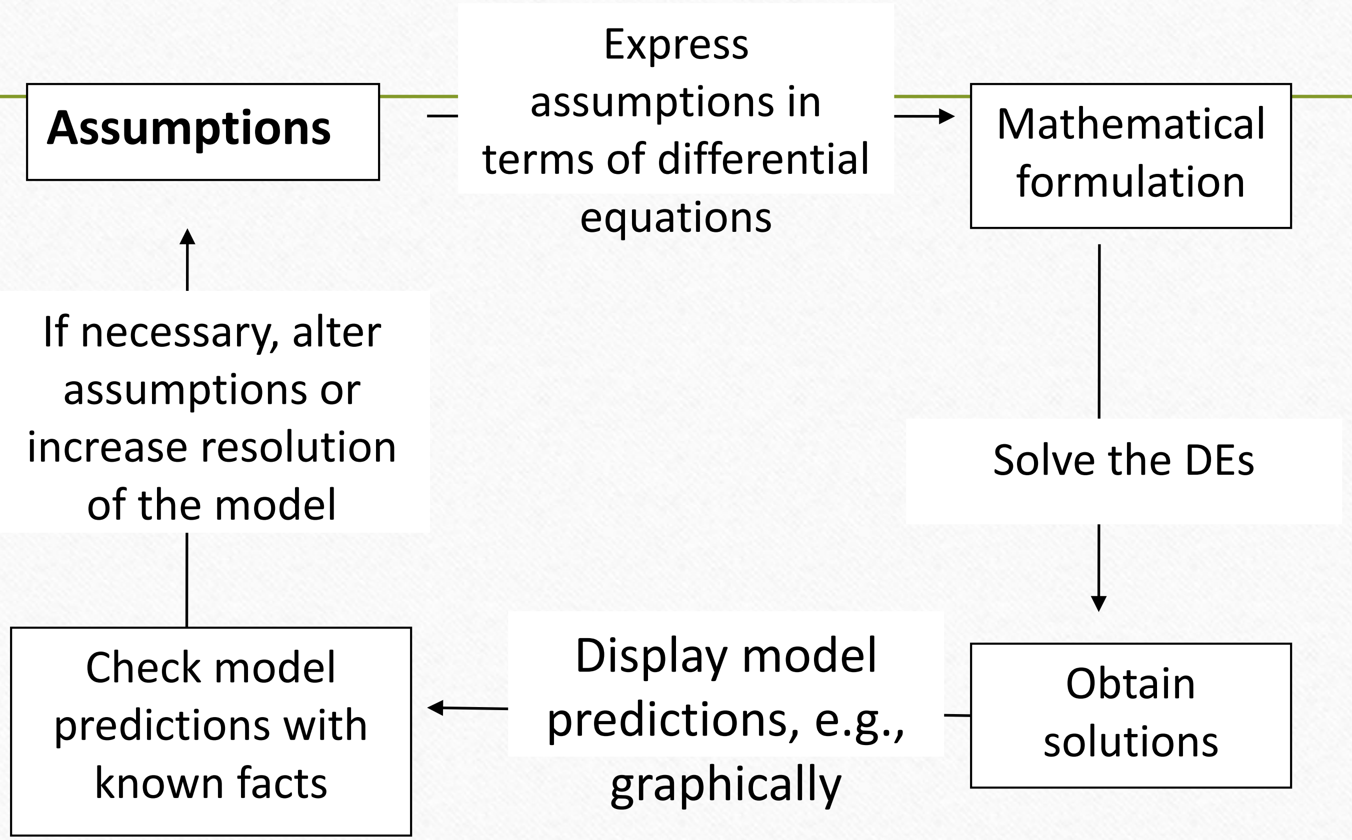
Some Basic Mathematical Models

Example 1.1

The population of the city of Yogyakarta increases at a rate proportional to the number of its inhabitants present at any time t. If the population of Yogyakarta was 30,000 in 1970 and 35,000 in 1980, what will be the population of Yogyakarta in 1990?


Assumptions
Express assumptions in terms of differential equations
Mathematical formulation
Solve the DEs
Obtain solutions
Display model predictions, e.g., graphically
Check model predictions with known facts
If necessary, alter assumptions or increase resolution of the model

Answer of example 1.1 :

Suppose $y(t)$ defines the number of population at time-*t*. Therefore, the first information can be transformed into mathematical model as

$$\frac{dy}{dt} = Ky$$

for some constant K.

$y(1970) = 30.000$ and $y(1980) = 35.000$

Definition and Terminology

Definition **Differential Equation**

An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a differential equation (DE).

- For Examples :

$$\frac{dy}{dx} = 2x$$

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$$

$$(x^2 + y^2)dx + (y - x)dy = 0$$

$$\left(\frac{dy}{dx}\right)^2 - 2y = 0$$

$$x\frac{dy}{dx} + (x + 1)y = x^3$$

Other examples of differential equation are

- **Radioactive Decay.** The rate at which the nuclei of a substance decay is proportional to the amount (more precisely, the number of nuclei) of the substance remaining at time t.

$$\frac{dA}{dt} = kA$$

- **Newton's Law of Cooling/warming**.

Suppose $T(t)$ represents the temperature of a body at time *t*, Tm the temperature of the surrounding medium. The rate at which the temperature of a body changes is proportional to the difference between the temperature of the body and the temperature of the surrounding medium.

$$\frac{dT}{dt} = k(T - T_m)$$