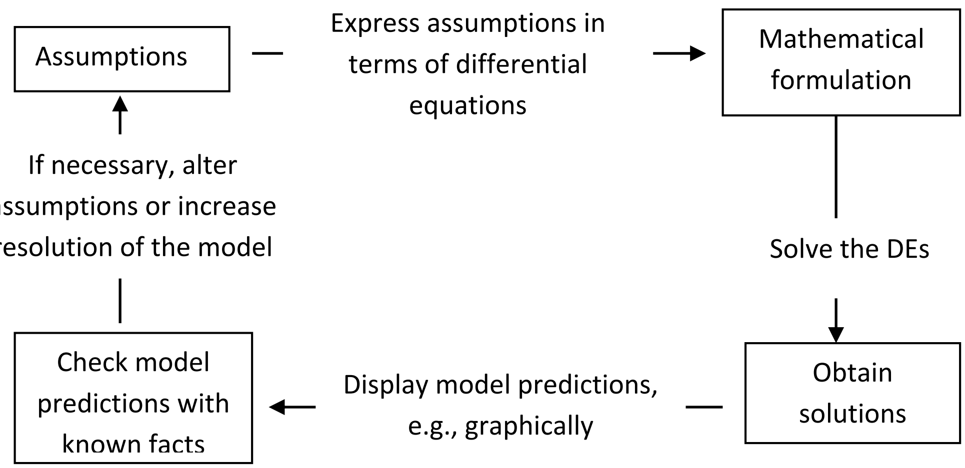
The study of differential equations has attracted the attention of many of the world's greatest mathematicians during the past three centuries. To give perspective to your study of differential equations, first, we use a problem to illustrate some of the basic ideas that we will return to and elaborate upon frequently throughout the remainder book.

1.1 Some Basic Mathematical Models

Consider the diagram below.



Example 1.1

The population of the city of Yogyakarta increases at a rate proportional to the number of its inhabitants present at any time t. If the population of Yogyakarta was 30,000 in 1970 and 35,000 in 1980, what will be the population of Yogyakarta in 1990?

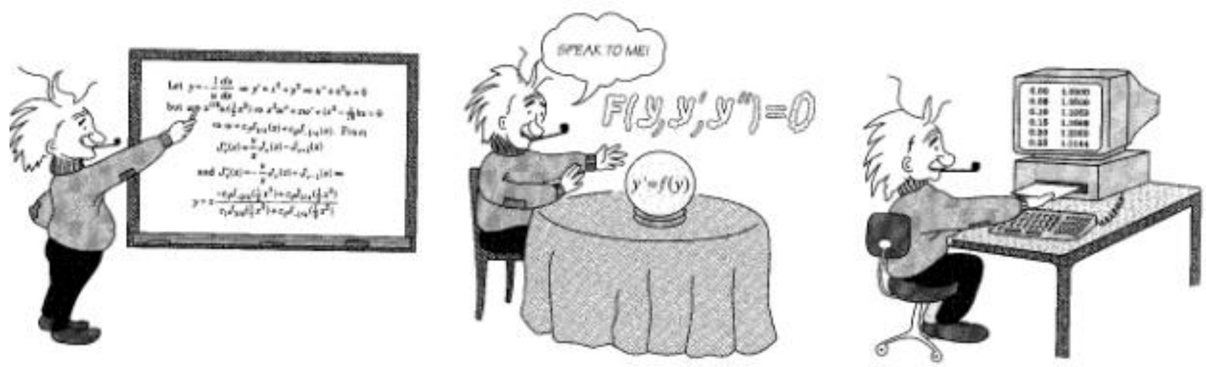
We can use the theory of differential equation to solve this problem. Suppose $y(t)$ defines the number of population at time-*t*. Therefore, the first information can be transformed into mathematical model as

$$\frac{dy}{dt} = Ky$$

for some constant *K*, $y(1970) = 30.000$, and $y(1980) = 35.000$.

Just what is a differential equation and what does it signify? Where and how do differential equations originate and of what use are they? Confronted with a differential equation, what does one do with it, how does one do it, and what are the results of such activity? These questions indicate three major aspects of the subject: theory, method, and application. Therefore, things that will be discussed in this handout are how the forms of differential equations are, the method to find its solution for each form and the last one is its applications.

When we talk about something that is constantly changing, for example rate, then I can say, in truth, we talk about derivative. Many of the principles underlying the behavior of the natural world are statement or relations involving rates at which things happen. When expressed in mathematical terms the relations are equations and the rates are derivatives. Equations containing derivatives are differential equations.



(a) analytical **(b)** qualitative **(c)** numerical

I.2 Definitions and Terminology

Definition **Differential Equation**

An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a differential equation (DE).

For examples

1. $\frac{dy}{dx} = 2x$
2. $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$
3. $\left(\frac{dy}{dx}\right)^2 - 2y = 0$
4. $(x^2 + y^2)dx + (y - x)dy = 0$
5. $x\frac{dy}{dx} + (x + 1)y = x^3$

Other examples of differential equation are

1. **Radioactive Decay.** The rate at which the nuclei of a substance decay is proportional to the amount (more precisely, the number of nuclei) $A(t)$ of the substance remaining at time t.
$$\frac{dA}{dt} = kA$$
2. **Newton's Law of Cooling/warming**. Suppose $T(t)$ represents the temperature of a body at time *t*, Tm the temperature of the surrounding medium. The rate at which the temperature of a body changes is proportional to the difference between the temperature of the body and the temperature of the surrounding medium.
$$\frac{dT}{dt} = k(T - T_m)$$

More examples can be found at page 20 – 24, A first course in Differential Equation with modeling application, Dennis G. Zill.

Exercises. 1

Population Dynamics.

1. Assumed that the growth rate of a country at a certain time is proportional to the total population of the country at that time $P(t)$. Determine a differential equation for the population of a country when individuals are allowed to immigrate into the country at a constant rate $r > 0$. How if individuals are allowed to emigrate from the country at a constant rate $r > 0$.
2. In another model of a changing population of a community, it is assumed that the rate at which the population changes is a net rate – that is, the difference between the rate of births and the rate of

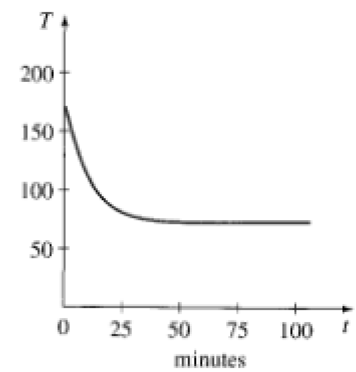
deaths in the community. Determine a model for the population $P(t)$ if both the birth rate and the death rate are proportional to the population present at time t.

3. Using the concept of net rate introduced in Problem 2, determine a model for a population $P(t)$ if the birth rate is proportional to the population present at time t but the death rate is proportional to the square of the population present at time t.
4. Modify the model in problem 3 for the net rate at which the population $P(t)$ of a certain kind of fish changes by also assuming that the fish are harvested at a constant rate $r > 0$.

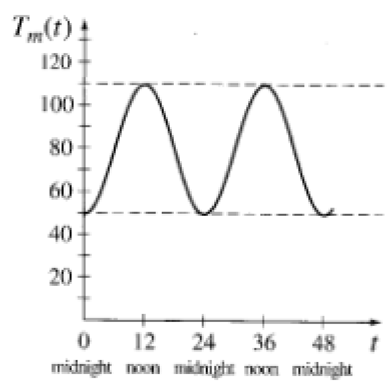
Newton's Law of Cooling/Warming

5. A cup of coffee cools according to Newtion's Law of cooling. Use data from the graph of the temperature $T(t)$ in figure beside to estimate the constants T_m, T_0 and k in a model of the form of a first-order initial-value problem

$$\frac{dT}{dt} = k(T - T_m), T(0) = T_0$$



6. The ambient temperature T_m could be a function of time t. Suppose that in an artificially controlled environment, T_m is periodic with a 24-hour period as illustrated in Figure beside. Devise a mathematical model for the temperature
$T(t)$ of a body within this environment.



Spread of a Disease/Technology

7. Suppose a student carrying in a flu virus returns to an isolated college campus of 1000 students. Determine a differential equation for the number of people $x(t)$ who have contracted the flu if the rate at which the disease spreads is proportional to the number of interaction between the number

of students who have the flu and the number of number of students who have the flu and the number of students who have not yet been exposed to it.

Mixtures

8. Supposes that a large mixing tank initially holds 300 gallons of water in which 50 pounds of salt have been dissolved. Pure water is pumped into the tank at a rate of 3 gal/min, and when the solution is well stirred, it is then pumped out at the same rate. Determine the differential equation for amount of salt $A(t)$ in the time at time t. What is $A(0)$.
9. Suppose that a large mixing tank initially holds 300 gallons of water in which 50 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 3 gal/min, and when the solution is well stirred, it is then pumped out at a slower rate 2 gal/min. If the concentration of the solution entering is 2 lb/gal, determine a differential equation for the amount of salt $A(t)$ in the tank at time t.

Falling bodies and Air Resistance

10. For high-speed motion through the air – such as the skydiver shown in figure below, falling before the paracute is opened – air resistance is closer to a power of the instantaneous velocity $v(t)$. Determine a differential equation for the velocity $v(t)$ of a falling body of mass m if air resistance is proportional to the square of the instantaneous velocity.