INTEGRAL OF CALCULUS

Arc Length (Intro)

Nikenasih B, M.Si
Mathematics Educational Department
Faculty of Mathematics and Natural Science
State University of Yogyakarta

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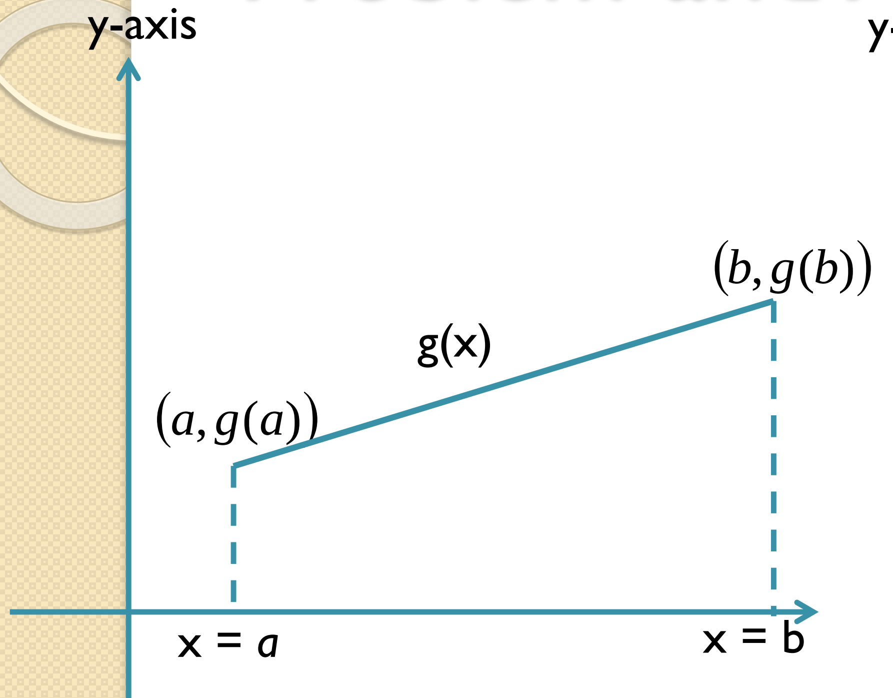
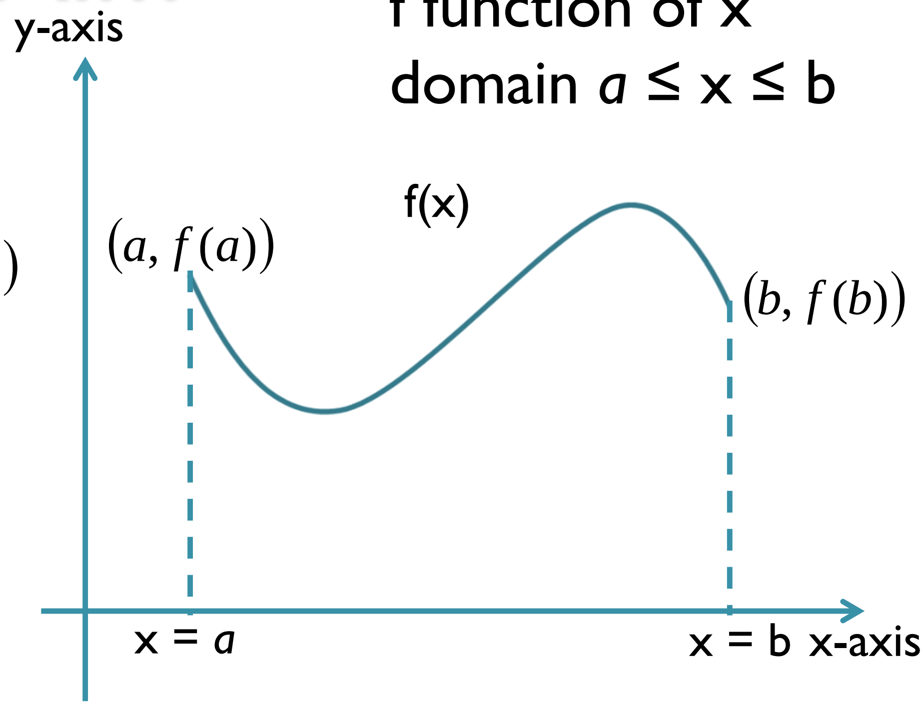
Problem and Aim



f function of x
domain $a \leq x \leq b$



$$d(A,B) = \sqrt{(g(b)-g(a))^2 + (b-a)^2}$$

What is the length of this arc?

Solution : A-R-L Method

- **What we Know**

  The length of a straight line

- **What we want to know**

  The volume of an arbitrary curve

- **How we do it**

  We approximate the curve with polygonal (broken) lines.

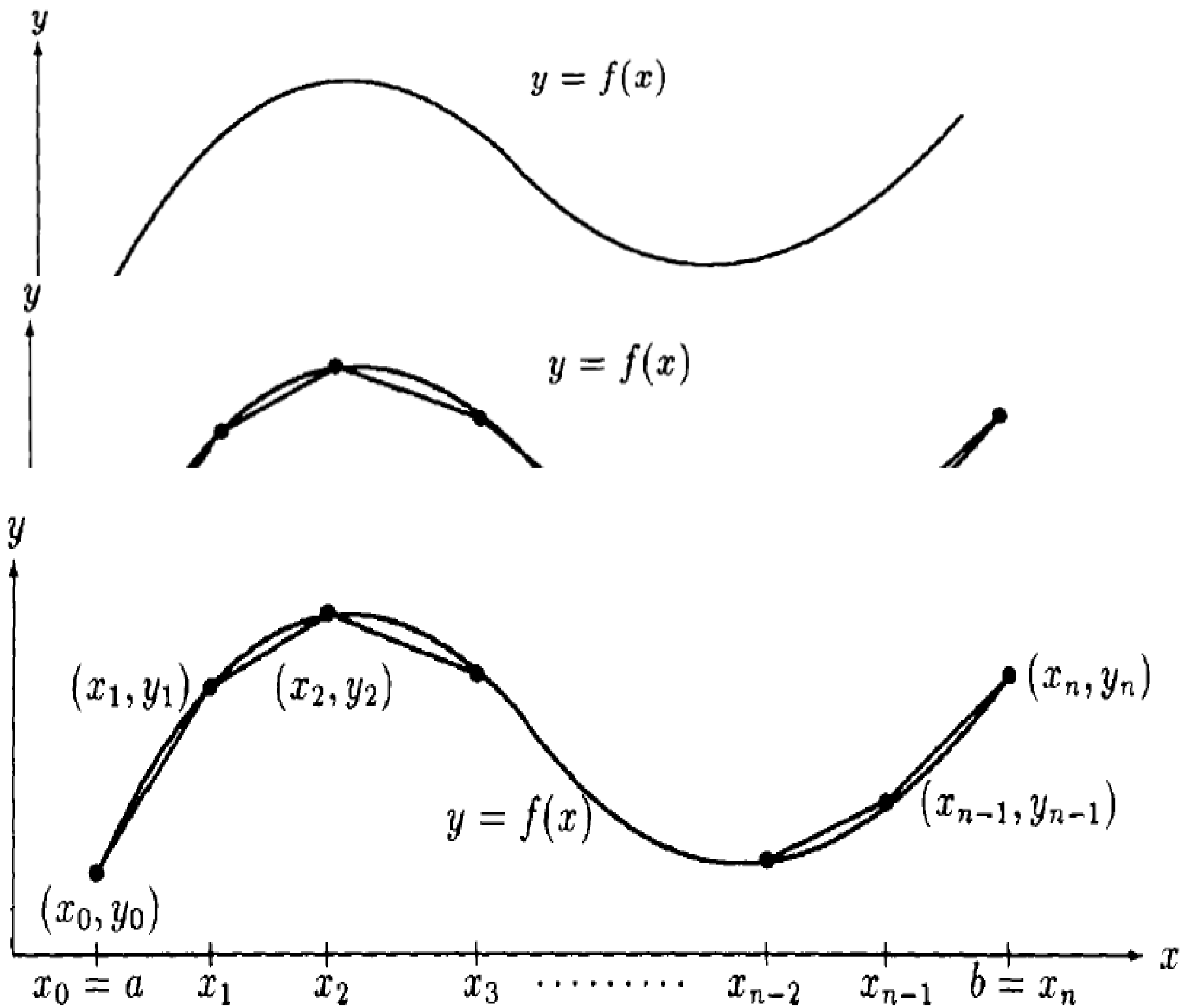


Figure 5-6: Initial approximation

The steps of solution (1)

- Approximate the region with n-straight lines, obtained by partitioning the interval [*a*,b]

$$x_0 = a, x_1, x_2, \cdots, x_n = b$$

  Here we get n-subinterval

$$[x_0 = a, x_1], \ [x_1, x_2], \ldots, [x_{n-1}, x_n = b]$$

- Determine points on the curve (x_0,y_0), (x_1,y_1), (x_2,y_2), ... , (x_n,y_n), where

$$y_i = f(x_i), \text{ for all } i = 1, 2, \ldots, n.$$

The steps of solution (2)

- Construct the line segments joining each consecutive pair of points, (x_0,y_0) with (x_1,y_1), (x_1,y_1) with (x_2,y_2), and so forth.
- Compute the length of this polygonal path. Use the formula of distance between two points to do this. Thus

The first segment length is $\sqrt{(y_1 - y_0)^2 + (x_1 - x_0)^2}$

The second segment length is $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

and so forth.

- Hence the total length of the polygon is

$$L_n = \sum_{i=1}^{n} \sqrt{\left(y_i - y_{i-1}\right)^2 + \left(x_i - x_{i-1}\right)^2}$$

$$L_n = \sum_{i=1}^{n} \left(x_i - x_{i-1}\right) \sqrt{\left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right)^2 + 1}$$

- Here we get

$$L_n = \sum_{i=1}^{n} \left(\Delta x\right) \sqrt{\left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right)^2 + 1}$$

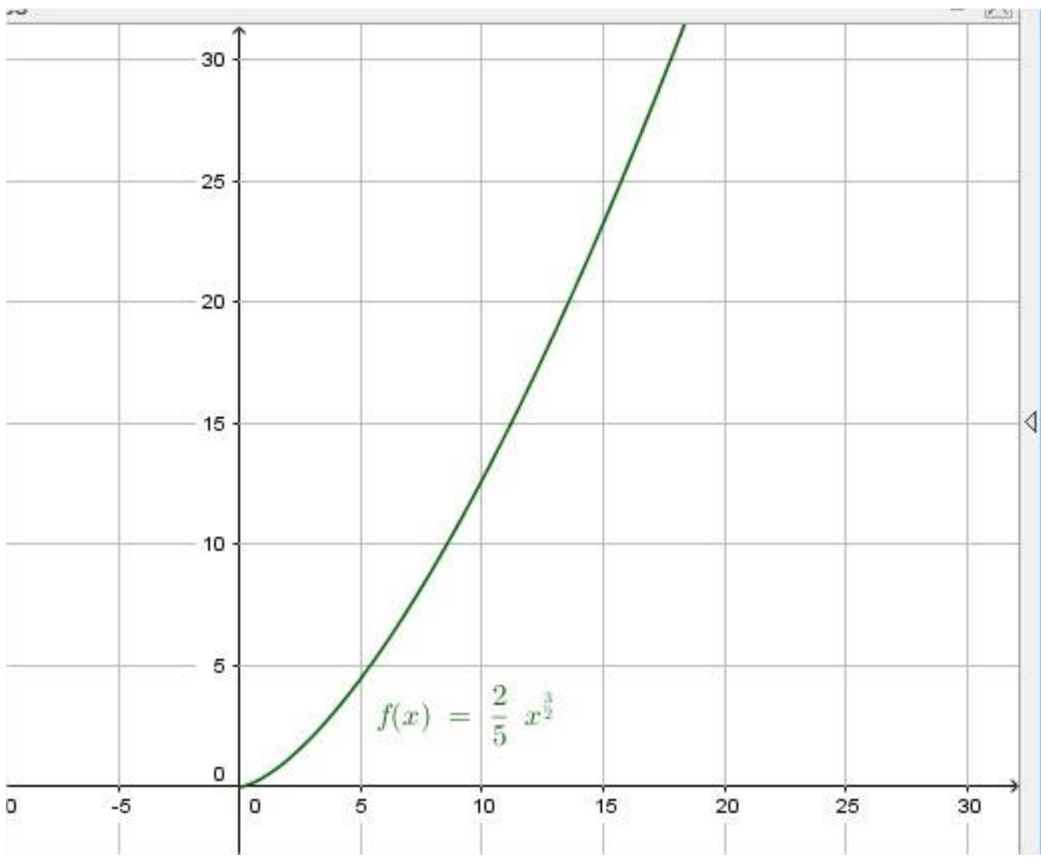
Kesimpulan

- Panjang kurva diperoleh untuk n →∞

$$L = \int_a^b \sqrt{\left(f'(x)\right)^2 + 1}\, dx$$

Example

Tentukan panjang kurva $y = \frac{2}{3}x^{\frac{3}{2}}, 0 \leq x \leq 15$.



Turunkan fungsi terhadap x

$$f'(x) = \frac{d}{dx}\left(\frac{2}{3}x^{\frac{3}{2}}\right) = x^{\frac{1}{2}}$$

Darisini diperoleh

$$L = \int_{0}^{15} \sqrt{\left(x^{0.5}\right)^2 + 1}\, dx = \int_{0}^{15} \sqrt{x+1}\, dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} \Big|_{0}^{15} = 10\sqrt{15}$$

Exercise

Tentukan panjang kurva $y = \frac{1}{3}(x^2 - 2)^{\frac{3}{2}}$ dari x = 3 sampai x = 6.