

INTEGRAL OF CALCULUS

Volume of a solid of a revolution (Intro)



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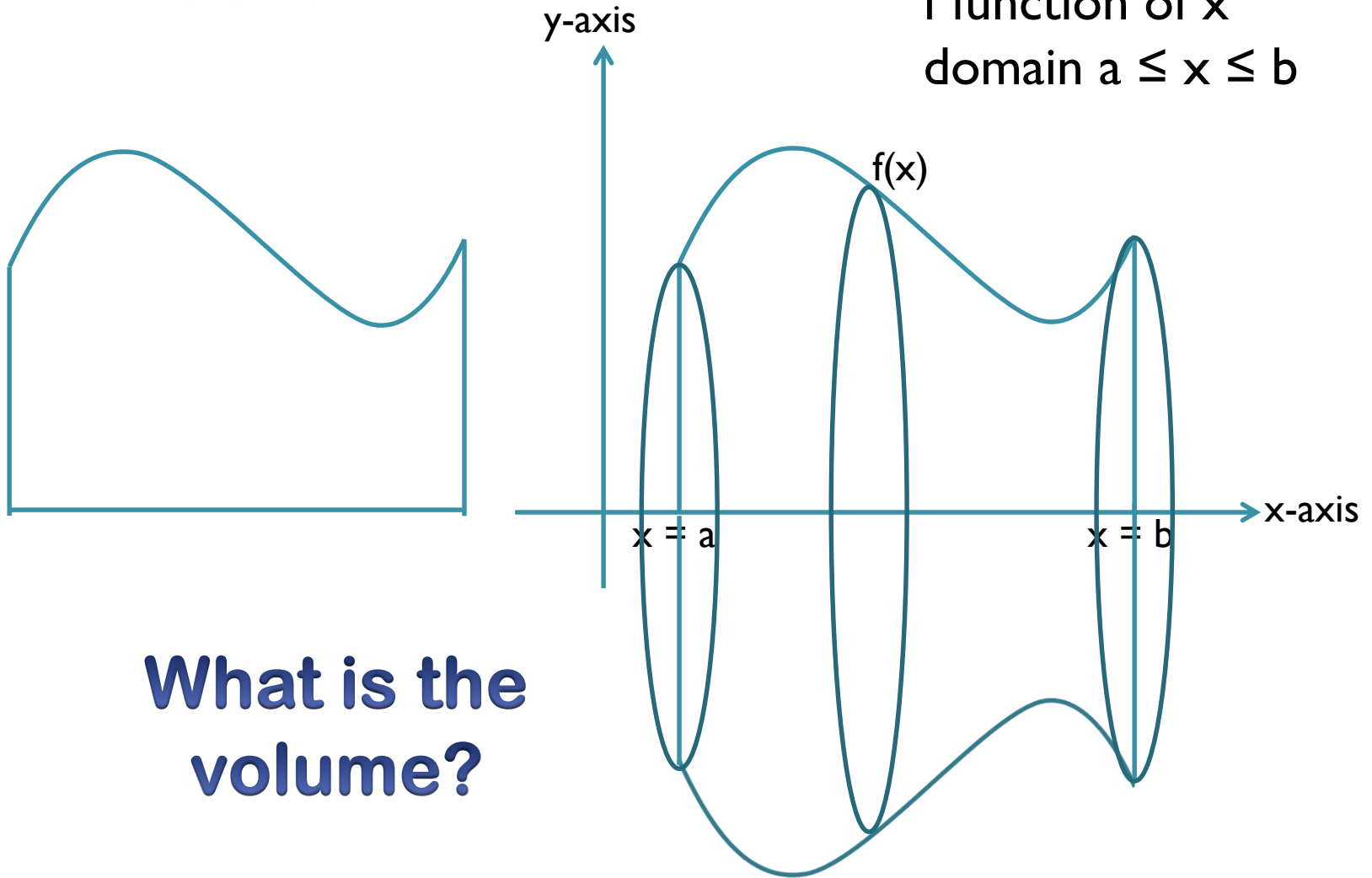


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Problem and Aim

f function of x
domain $a \leq x \leq b$



**What is the
volume?**

Solution :A-R-L Method

- **What we Know**

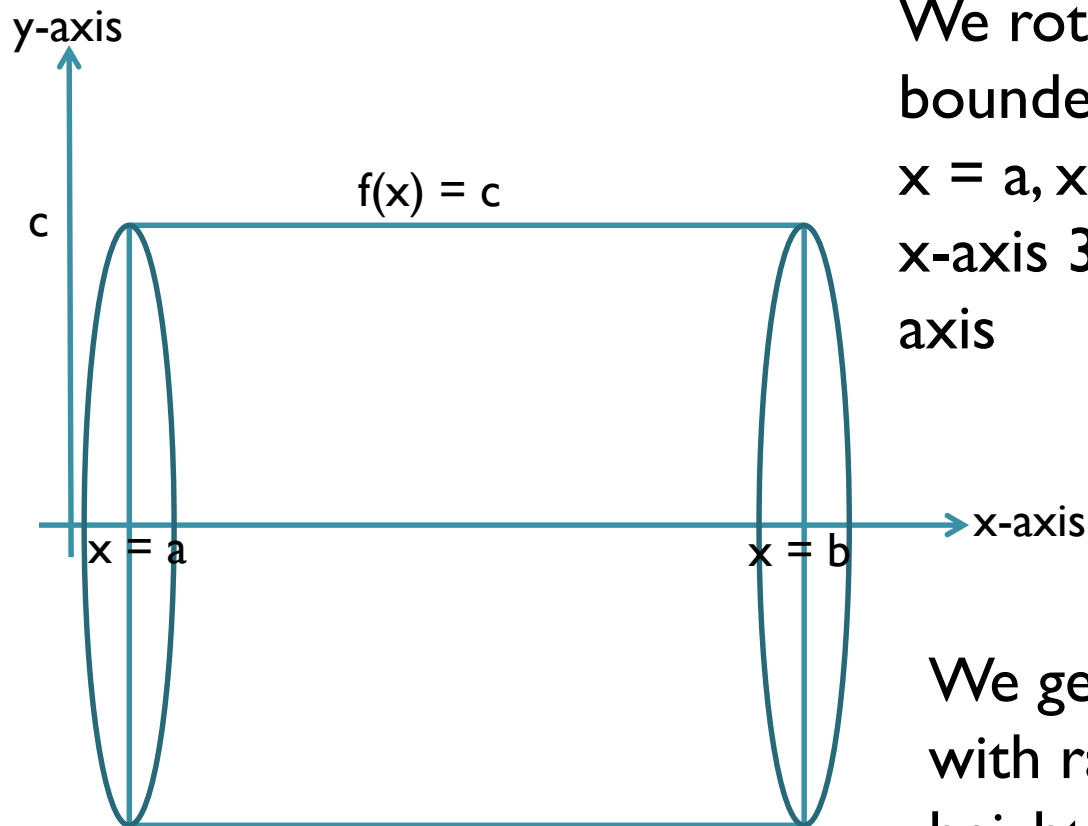
The volume of a cylinder

- **What we want to know**

The volume of an arbitrary solid of a revolution

- **How we do it**

We approximate the solid with a certain number of cylinders



We rotate region bounded by $f(x) = c$, $x = a$, $x = b$ and x-axis 360° about x-axis

We get cylinder, with radius c and height $b - a$.

$$V = \pi c^2 (b - a)$$

The steps of solution (I)

- Approximate the region with n-rectangle, obtained by partitioning the interval $[a,b]$

$$x_0 = a, x_1, x_2, \dots, x_n = b$$

Here we get n-subinterval

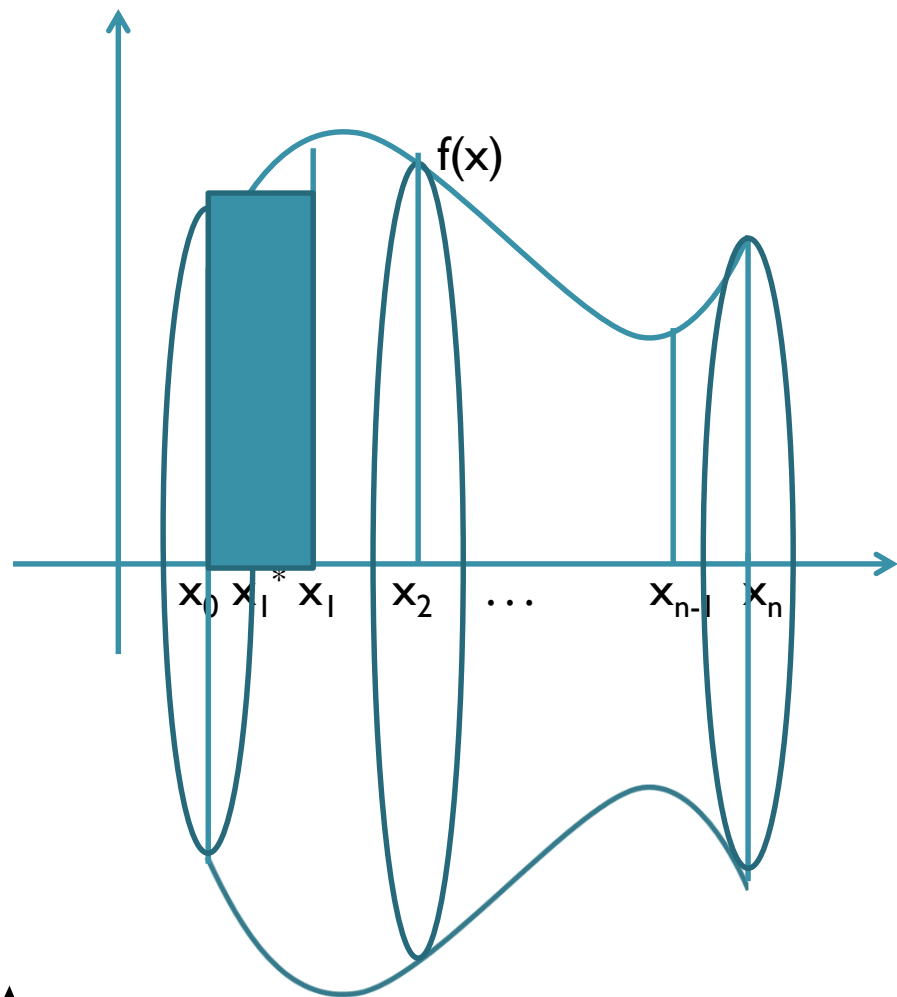
$$[x_0 = a, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n = b]$$

- Select a point at each subinterval, suppose x_i^* for i^{th} -subinterval

Here we get n-rectangles where i^{th} -rectangle has width $x_i - x_{i-1}$ and length $f(x_i^*)$

The steps of solution (2)

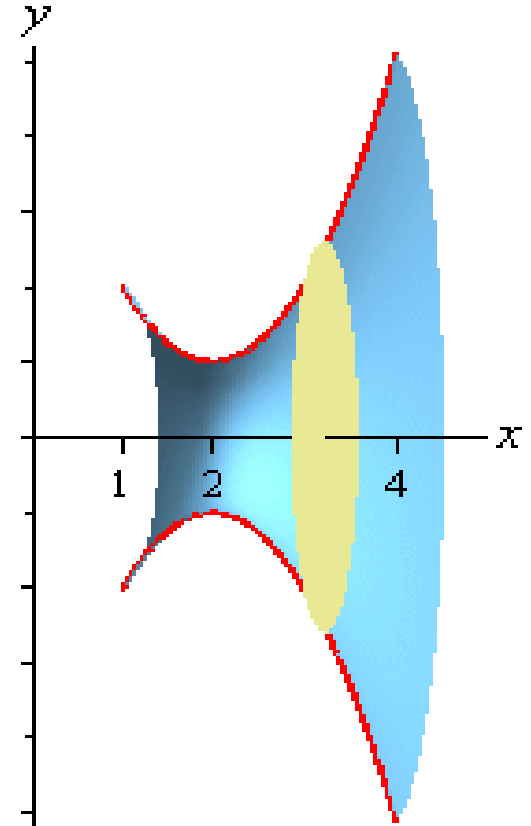
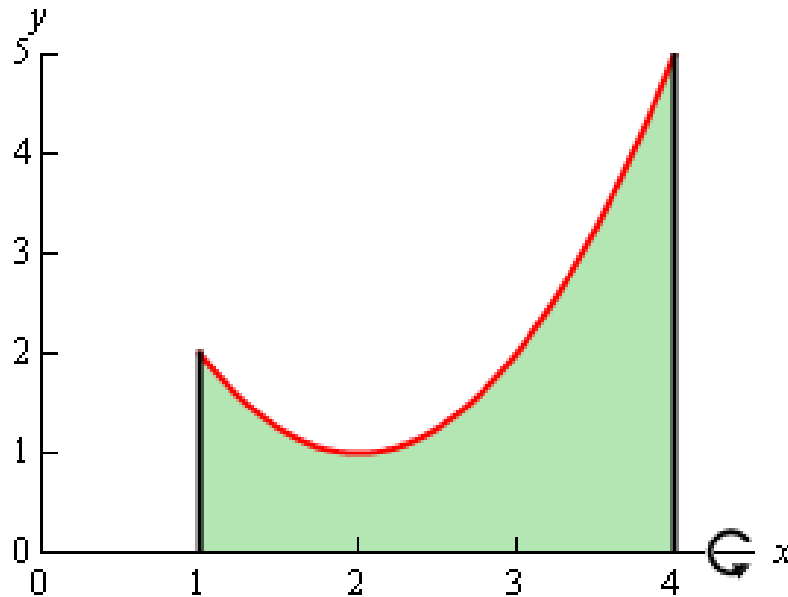
- By rotating the rectangle, we get n -cylinders where i^{th} -cylinder has radius $f(x_i^*)$ and height $(x_i - x_{i-1})$
- The volume of a solid of a revolution can be approximated by the sum volume of n -cylinders.



$$V \approx \pi \sum_{i=1}^n f^2(x_i^*) \Delta x$$

Examples (I)

- Approximate the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$ and the x-axis about the x-axis

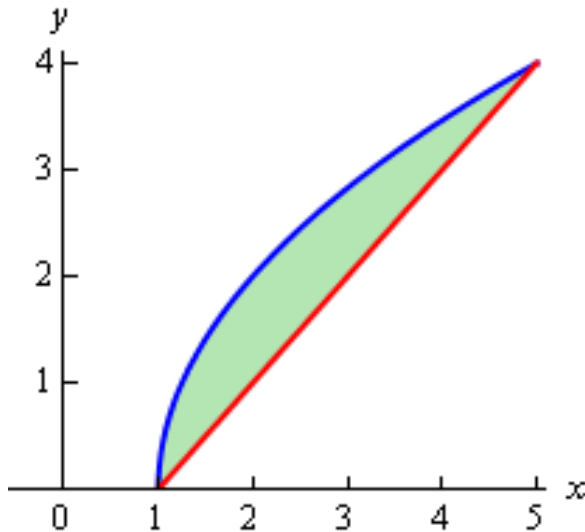




Answer (I)

Examples (2)

- Approximate the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x-1$ about the x-axis





Answer (2)

Exercises

- Approximate the volume of the solid obtained by rotating the region bounded by $y = \sqrt[3]{x}$, $x = 8$ and the x-axis about the x-axis