INTEGRAL OF CALCULUS

Arc Length (Intro)

Nikenasih B, M.Si
Mathematics Educational Department
Faculty of Mathematics and Natural Science
State University of Yogyakarta
Contents

• Problem and Aim
• Solution : A-R-L Method
• The Steps of Solution
• Examples
• Exercises
Problem and Aim

The domain of the function $f(x)$ is $a \leq x \leq b$.

What is the length of this arc?
Solution: A-R-L Method

• What we Know
  The length of a straight line

• What we want to know
  The volume of an arbitrary curve

• How we do it
  We approximate the curve with polygonal (broken) lines.
Figure 5.6: Initial approximation
The steps of solution (1)

- Approximate the region with n-straight lines, obtained by partitioning the interval \([a, b]\)
  \[x_0 = a, x_1, x_2, \ldots, x_n = b\]
  Here we get n-subinterval
  \([x_0 = a, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n = b]\)
- Determine points on the curve \((x_0, y_0), (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), where
  \[y_i = f(x_i), \text{ for all } i = 1, 2, \ldots, n.\]
The steps of solution (2)

- Construct the line segments joining each consecutive pair of points, \((x_0, y_0)\) with \((x_1, y_1)\), \((x_1, y_1)\) with \((x_2, y_2)\), and so forth.
- Compute the length of this polygonal path. Use the formula of distance between two points to do this. Thus

The first segment length is \(\sqrt{(y_1 - y_0)^2 + (x_1 - x_0)^2}\)

The second segment length is \(\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}\)

and so forth.
- Hence the total length of the polygon is

\[ L_n = \sum_{i=1}^{n} \sqrt{(y_i - y_{i-1})^2 + (x_i - x_{i-1})^2} \]

\[ L_n = \sum_{i=1}^{n} (x_i - x_{i-1}) \sqrt{\left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right)^2} + 1 \]

- Here we get

\[ L_n = \sum_{i=1}^{n} (\Delta x) \sqrt{\left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right)^2} + 1 \]
Kesimpulan

- Panjang kurva diperoleh untuk $n \to \infty$

\[ L = \int_{a}^{b} \sqrt{\left(f'(x)\right)^2 + 1} \, dx \]
Example

Tentukan panjang kurva \( y = \frac{2}{3} x^{\frac{3}{2}} \), \( 0 \leq x \leq 15 \).
Turunkan fungsi terhadap x

\[ f'(x) = \frac{d}{dx} \left( \frac{2}{3} x^{\frac{3}{2}} \right) = x^{\frac{1}{2}} \]

Darisini diperoleh

\[ L = \int_{0}^{15} \sqrt{\left(x^{0.5}\right)^2 + 1} \, dx = \int_{0}^{15} \sqrt{x + 1} \, dx \]

\[ = \frac{2}{3} x^{\frac{3}{2}} \bigg|_{0}^{15} = 10\sqrt{15} \]
Exercise

Tentukan panjang kurva \( y = \frac{1}{3} (x^2 - 2)^{\frac{3}{2}} \)
dari \( x = 3 \) sampai \( x = 6 \).